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TURBULENT FREE-CONVECTIVE JETS: NUMERICAL SOLUTION OF MODEL EQUATIONS OF TRANSFER

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Numerical solutions of self-similar equations for plane and axisymmetric free-convective jets are obtained using different semiempirical turbulence models. The results obtained are compared to the experimental data of other authors.

Introduction. Turbulent free-convective flows induced by buoyancy sources are the subject matter of a great number of recent publications in the technical literature, which is governed to a considerable degree by their direct relation to diverse modern problems of science and technology. Knowledge of the special features and regularities of such problems is needed, for instance, to solve problems dealing with air conditioning, evaluation of the degree of environmental contamination under the action of anthropogenic factors, fire problems, a search for methods of efficient reburning of environmentally hazardous gases, and so on. From the fundamental viewpoint, the problem of mathematical modeling of such jet flows is not conclusively resolved. The nonlinear character of interaction of scalar and vector fields makes a description of the sought characteristics more complicated by an order of magnitude and allows this phenomenon in the heat-transfer theory to be surely assigned to the most complicated ones. The latter was the reason for the fact that, despite the simplicity of differential equations, which describe the behavior of free-convective flows within the framework of various turbulent models, construction of solutions of these equations turned out to be a difficult matter. Therefore, the statement of problems was subjected to further simplifications that entailed loss of quantitative information. Naturally, such approaches did not allow a sufficiently accurate determination of the advantages and drawbacks of a particular mathematical problem. The situation becomes more complicated due to the fact that the presently known experimental data on free-convective jets [1-10] have a substantial scatter not only in the pulsation characteristics of the flow but also in the averaged parameters. Under these circumstances, investigations concerned with evaluation of the possibilities of different mathematical turbulence models to describe and predict the development of free-convective jets are, undoubtedly, of interest in our opinion.

Below, we provide results of a complex numerical solution of a class of self-similar problems on plane and axisymmetric free-convective jet flows obtained using two different semiempirical turbulence models.

Basic Equations. An initial system of partial differential equations for the averaged parameters of a vertical free-convective jet in the approximation of the theory of a turbulent boundary layer is of the form

$$\frac{\partial}{\partial x}(y^j u) + \frac{\partial}{\partial y}(y^j v) = 0 ,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{y^j}\frac{\partial}{\partial y}(y^j \langle u'v'\rangle) + g\beta\Delta T ,$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -\frac{1}{y^j}\frac{\partial}{\partial y}(y^j \langle v'T'\rangle) . \quad (1)$$

To close equalities (1), in the present work use is made of the Boussinesq hypothesis

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$$-\langle u'v'\rangle = \nu_t \frac{\partial u}{\partial y}, \quad -\langle v'T'\rangle = \frac{\nu_t}{\sigma_t}\frac{\partial T}{\partial y} \tag{2}$$

and of additional semiempirical relations for the coefficient of turbulent viscosity ν_t:

$$\nu_t = \kappa_0 b\,(u_{\max} - u_{\min})\,; \tag{3}$$

$$\nu_t = \kappa_0 \frac{x}{y^{j+1}} \int_0^y u y^j dy\,. \tag{4}$$

Next, from the conditions of symmetry and locality of the phenomenon, the boundary conditions follow for the velocity and temperature fields:

$$y = 0: \quad v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0\,;$$
$$y \to \infty: \quad y \to 0\,, \quad T \to T_\infty\,. \tag{5}$$

The statement of the problem is completed with the representation of the integral law of conservation of the heat flux

$$Q_0 = 2\,(\pi)^j \int_0^\infty \rho C_p\, u \Delta T y^j dy = \text{const}\,, \tag{6}$$

which stems from the third equation of system (1).

Plane Jet ($j = 0$). Analyzing the development of a jet flow over the main (self-similar) section of flow, we assume that

$$u = \left(\frac{g\beta Q_0}{\rho C_p \kappa_0}\right)^{1/3} f'(\eta)\,, \quad \nu_t = \left(\frac{g\beta Q_0 \kappa_0^5}{\rho C_p}\right)^{1/3} x\,,$$
$$\Delta T = \left(\frac{Q_0}{\rho C_p \sqrt{g\beta}\;\kappa_0}\right)^{2/3} h(\eta)\, x^{-1}\,, \quad \eta = \frac{y}{\kappa_0 x}\,. \tag{7}$$

Then for finding the unknown functions $f(\eta)$ and $h(\eta)$ we obtain the following system of interrelated ordinary differential equations, where differentiation with respect to a new variable η is denoted by a prime:

$$f''' + ff'' + h = 0\,; \quad \frac{1}{\sigma_t} h'' + fh' + f'h = 0\,;$$
$$f(0) = 0\,, \quad f''(0) = 0\,, \quad f'(\infty) = 0\,, \quad h'(0) = 0\,, \quad h(\infty) = 0\,; \tag{8}$$
$$\int_0^\infty f' h\, d\eta = \frac{1}{2}\,.$$

Note that problem (8) allows integration by quadratures only for two particular cases [11]:

$$\sigma_t = \frac{2}{3}: \ f' = 3\alpha^2 (1 - \tanh^2 \alpha\eta) \, , \ h = 6\alpha^4 (1 - \tanh^2 \alpha\eta) \, , \ \alpha = \left(\frac{1}{24}\right)^{1/5} ;$$

$$\sigma_t = 2: \ f' = 2\alpha^2 (1 - \tanh^2 \alpha\eta) \, , \ h = 4\alpha^4 (1 - \tanh^2 \alpha\eta)^2 \, , \ \alpha = \frac{1}{2}\left(\frac{15}{4}\right)^{1/5} . \quad (9)$$

According to formula (3) (the second Prandtl hypothesis), the coefficient of turbulent viscosity is ν_t = const at x = const. The latter, naturally, much idealizes the physics of the process investigated. Therefore, expression (4), suggested for the first time in [12, 13], is more perfect; its use in calculations leads to the fact that the behavior of the parameter

$$\nu_t = \left(\frac{g\beta Q_0 \kappa_0^5}{\rho C_p}\right)^{1/3} \frac{f(\eta)}{\eta} x \, , \quad (10)$$

and of other characteristics, i.e., u, v, and ΔT, is found from solving a two-point boundary-value problem of the form

$$\left(\frac{f}{\eta} f''\right)' + ff'' + h = 0 \, ; \ \frac{1}{\sigma_t}\left(\frac{f}{\eta} h'\right)' + fh' + f'h = 0 \, ;$$

$$f(0) = 0 \, , \ f''(0) = 0 \, , \ f'(\infty) = 0 \, , \ h'(0) = 0 \, , \ h(\infty) = 0 \, ; \quad (11)$$

$$\int_0^\infty f' h d\eta = \frac{1}{2} .$$

At $\sigma_t = 2$, system (11) has the analytical solution [14]

$$f' = \alpha \exp\left(-\frac{\eta^2}{2}\right), \ h = \alpha^2 \exp(-\eta^2) \, , \ \alpha = \left(\frac{3}{2\pi}\right)^{1/6} . \quad (12)$$

Next, using the scales from [15] for normalization of the results obtained, we can represent formulas (7) in dimensionless form:

$$u_c = \frac{f'(0)}{\sqrt[3]{\kappa_0}} F_0^{1/3} = A_u F_0^{1/3} \, , \ \Delta T_c = \frac{h(0)}{\sqrt[3]{\kappa_0^2}} F_0^{-1/3} x^{-1} = A_T F_0^{-1/3} x^{-1} \, ,$$

$$y_{0.5u} = \kappa_0 \eta_{0.5u} x \, , \ y_{0.5\Delta T} = \kappa_0 \eta_{0.5\Delta T} x \, , \quad (13)$$

$$\frac{\langle u'v' \rangle_m}{u_c^2} = \kappa_0 (uv)_m \, , \ \frac{\langle v'T' \rangle_m}{u_c \Delta T_c} = \kappa_0 (vT)_m \, .$$

Axisymmetric Jet (j = 1). The scheme of calculation of the main section of a vertical axisymmetric free-convective flow is identical to that of a plane jet. A solution of system (1)-(3), (5), and (6) can be represented in the form

$$u = \left(\frac{g\beta Q_0}{4\pi\rho C_p \kappa_0}\right)^{1/3} f'(\eta) x^{-1/3} \, , \ \nu_t = \left(\frac{g\beta Q_0 \kappa_0^2}{4\pi\rho C_p}\right)^{1/3} x^{2/3} \, ,$$

$$\Delta T = \left(\frac{Q_0}{4\pi\rho C_p \sqrt{g\beta} \ \kappa_0}\right)^{2/3} h(\eta) x^{-5/3} \, , \ \eta = \frac{y^2}{4\kappa_0 x^2} . \quad (14)$$

Then we arrive at the necessity of integration of two nonlinear ordinary differential equations:

$$(\eta f'')' + \frac{5}{3} f f'' + \frac{1}{3} f'^2 + h = 0 \, ; \quad \frac{1}{\sigma_t} (\eta h')' + \frac{5}{3} f h' + \frac{5}{3} f' h = 0 \, ;$$

$$f(0) = 0 \, , \quad \lim_{\eta \to 0} \sqrt{\eta} \, f'' = 0 \, , \quad f'(\infty) = 0 \, ; \tag{15}$$

$$\lim_{\eta \to 0} \sqrt{\eta} \, h' = 0 \, , \quad h(\infty) = 0 \, ; \quad \int_0^\infty f' h d\eta = 1 \, .$$

At $\sigma_t = 1.1$ and $\sigma_t = 2$, problem (15) is solved analytically in [11]:

$$\sigma_t = 1.1 : f' = \frac{18}{11} \frac{\alpha}{(1 + \alpha\eta)^2} \, , \quad h = \frac{288}{121} \frac{\alpha^2}{(1 + \alpha\eta)^3} \, , \quad \alpha = \frac{11\sqrt{11}}{36} \, ;$$

$$\sigma_t = 2 : f' = \frac{6}{5} \frac{\alpha}{(1 + \alpha\eta)^2} \, , \quad h = \frac{48}{25} \frac{\alpha^2}{(1 + \alpha\eta)^4} \, , \quad \alpha = \frac{25\sqrt{2}}{24} \, . \tag{16}$$

The use of hypothesis (4) in modeling of the problem under investigation allows us to seek a solution among functions of the form

$$u = \left(\frac{g\beta Q_0}{8\pi\rho C_p \kappa_0} \right)^{1/3} f'(\eta) \, x^{-1/3} \, , \quad \nu_t = \left(\frac{g\beta Q_0 \kappa_0^2}{\pi\rho C_p} \right)^{1/3} \frac{f(\eta)}{\eta} x^{2/3} \, ,$$

$$\Delta T = \frac{1}{2} \left(\frac{Q_0}{\pi\rho C_p \sqrt{g\beta} \, \kappa_0} \right)^{2/3} h(\eta) \, x^{-5/3} \, , \quad \eta = \frac{y^2}{4\kappa_0 x^2} \, . \tag{17}$$

By substituting expressions (17) into the initial system of equations, we arrive at

$$(f f'')' + \frac{5}{6} f f'' + \frac{1}{6} f'^2 + h = 0 \, ; \quad \frac{1}{\sigma_t} (f h')' + \frac{5}{6} f h' + \frac{5}{6} f' h = 0 \, ;$$

$$f(0) = 0 \, , \quad \lim_{\eta \to 0} \sqrt{\eta} \, f'' = 0 \, , \quad f'(\infty) = 0 \, ; \tag{18}$$

$$\lim_{\eta \to 0} \sqrt{\eta} \, h' = 0 \, , \quad h(\infty) = 0 \, ; \quad \int_0^\infty f' h \, d\eta = 1 \, .$$

Problem (18) is considered for the first time in [12], where an analytical solution is obtained for particular values of the turbulent Prandtl number:

$$\sigma_t = 0.6 : f' = \alpha \exp\left(-\frac{\eta}{2} \right) , \quad h = \frac{1}{3} \alpha^2 \exp\left(-\frac{\eta}{2} \right) , \quad \alpha = (3)^{1/3} \, ;$$

$$\sigma_t = 2 : f' = \alpha \exp\left(-\frac{5}{6} \eta \right) , \quad h = \frac{2}{3} \alpha^2 \exp\left(-\frac{5}{3} \eta \right) , \quad \alpha = \left(\frac{15}{4} \right)^{1/3} . \tag{19}$$

With allowance for the scales from [15], formulas (14) can be rewritten in the form

TABLE 1. Values of the Parameters $f'(0)$, $h(0)$, $\eta_{0.5u}$, $\eta_{0.5\Delta T}$, $(uv)_m$, and $(vT)_m$ for a Plane Free-Convective Jet

σ_t	$f'(0)$		$h(0)$		$\eta_{0.5u}$		$\eta_{0.5\Delta T}$		$(uv)_m$		$(vT)_m$	
	(8)	(11)	(8)	(11)	(8)	(11)	(8)	(11)	(8)	(11)	(8)	(11)
0.4	0.8375	–	0.3914	–	1.859	–	2.198	–	0.4306	–	0.9092	–
0.5	0.8395	0.8766	0.4247	0.4839	1.769	1.482	1.945	1.665	0.4539	0.4201	0.8250	0.7254
0.6	0.8408	0.8773	0.4541	0.5134	1.701	1.434	1.761	1.520	0.4733	0.4356	0.7614	0.6731
2/3	0.8414	0.8777	0.4721	0.5315	1.664	1.407	1.664	1.442	0.4846	0.4443	0.7267	0.6441
0.7	0.8417	0.8779	0.4807	0.5402	1.647	1.395	1.621	1.407	0.4897	0.4483	0.7111	0.6306
0.7187	–	0.8780	–	0.5450	–	1.389	–	1.389	–	0.4504	–	0.6240
0.8	0.8424	0.8784	0.5052	0.5648	1.603	1.363	1.509	1.316	0.5038	0.4595	0.6699	0.5961
0.9	0.8430	0.8789	0.5279	0.5878	1.566	1.336	1.417	1.241	0.5161	0.4695	0.6353	0.5670
1.0	0.8436	0.8794	0.5494	0.6093	1.534	1.313	1.340	1.778	0.5269	0.4778	0.6057	0.5410

TABLE 2. Values of the Parameters $f'(0)$, $h(0)$, $\eta_{0.5u}$, $\eta_{0.5\Delta T}$, $(uv)_m$, and $(vT)_m$ for an Axisymmetric Free-Convective Jet

σ_t	$f'(0)$		$h(0)$		$\eta_{0.5u}$		$\eta_{0.5\Delta T}$		$(uv)_m$		$(vT)_m$	
	(15)	(18)	(15)	(18)	(15)	(18)	(15)	(18)	(15)	(18)	(15)	(18)
0.4	1.4711	1.4088	0.9889	0.5313	0.777	1.643	0.866	2.080	0.2524	0.2268	0.6030	0.4857
0.4785	1.5062	–	1.1631	–	0.693	–	0.693	–	0.2619	–	0.5528	–
0.5	1.5147	1.4272	1.2101	0.6147	0.674	1.497	0.657	1.664	0.2644	0.2386	0.5411	0.4456
0.6	1.5492	1.4422	1.4256	0.6934	0.599	1.387	0.526	1.387	0.2749	0.2486	0.4949	0.4145
0.7	1.5776	1.4549	1.6360	0.7688	0.543	1.299	0.437	1.189	0.2843	0.2575	0.4588	0.3895
0.8	1.6018	1.4662	1.8424	0.8414	0.499	1.227	0.373	1.040	0.2928	0.2652	0.4294	0.3686
0.9	1.6229	1.4764	2.0455	0.9119	0.463	1.667	0.324	0.925	0.3004	0.2721	0.4050	0.3510
1.0	1.6417	1.4857	2.2461	0.9805	0.434	1.116	0.286	0.832	0.3074	0.2783	0.3842	0.3357
1.1	1.6586	–	2.4445	–	0.408	–	0.256	–	0.3139	–	0.3662	–

$$u_c = \frac{f'(0)}{\sqrt[3]{16\kappa_0}} F_0^{-1/3} x^{-1/3} = A_u F_0^{-1/3} x^{-1/3}, \quad \Delta T_c = \frac{h(0)}{\sqrt[3]{256\kappa_0^2}} F_0^{1/3} x^{-5/3} = A_T F_0^{1/3} x^{-5/3},$$

$$y_{0.5u} = 2\sqrt{\kappa_0}\sqrt{\eta_{0.5u}}\, x, \quad y_{0.5\Delta T} = 2\sqrt{\kappa_0}\sqrt{\eta_{0.5\Delta T}}\, x, \tag{20}$$

$$\frac{\langle u'v'\rangle_m}{u_c^2} = \sqrt{\kappa_0}\,(uv)_m, \quad \frac{\langle v'T'\rangle_m}{u_c \Delta T_c} = \sqrt{\kappa_0}\,(vT)_m.$$

Similarly, for equalities (17) we have

$$u_c = \frac{f'(0)}{\sqrt[3]{32\kappa_0}} F_0^{-1/3} x^{-1/3} = A_u F_0^{-1/3} x^{-1/3},$$

$$\Delta T_c = \frac{h(0)}{\sqrt[3]{128\kappa_0^2}} F_0^{1/3} x^{-5/3} = A_T F_0^{1/3} x^{-5/3},$$

$$y_{0.5u} = 2\sqrt{\kappa_0}\sqrt{\eta_{0.5u}}\, x, \quad y_{0.5\Delta T} = 2\sqrt{\kappa_0}\sqrt{\eta_{0.5\Delta T}}\, x, \tag{21}$$

$$\frac{\langle u'v'\rangle_m}{u_c^2} = 2\sqrt{\kappa_0}\,(uv)_m, \quad \frac{\langle v'T'\rangle_m}{u_c \Delta T_c} = 2\sqrt{\kappa_0}\,(vT)_m.$$

TABLE 3. Structure of Turbulent Free-Convective Jets

Ref.	Jet	σ_t	A_u	A_T	$y_{0.5u}/x$	$y_{0.5\Delta T}/x$	$\frac{\langle u'v'\rangle_m}{u_c^2}$	$\frac{\langle v'T'\rangle_m}{u_c \Delta T_c}$
Experimental data								
[1]	Plane	–	1.80	2.60	0.147	0.130	–	–
[2]	Same	–	1.66	2.38	0.097	0.130	–	–
[6]	Same	–	2.05	3.20	0.130	0.128	–	–
[8]	Same	0.65–0.90	–	–	0.130	0.130	0.026	0.033
[9]	Same	0.46–0.70	2.13	2.56	0.110	0.133	0.031	0.045
Calculation								
Our data, problem (8)	Plane	0.6	2.0	2.58	0.126	0.130	0.035	0.056
Our data, problem (11)	Same	0.6	1.99	2.65	0.123	0.130	0.037	0.058
[2]	Same	var	2.0	3.0	0.105	0.113	0.030	0.046
Experimental data								
[1]	Circular	–	4.7	11.0	0.085	0.099	–	–
[4]	Same	–	–	–Á[Ç[À–Â	0.133	0.105	–	–
[3]	Same	–	3.4	9.1	0.112	0.103	–	–
[9]	Same	0.95	–	–	–	–	0.024	0.032
[5]	Same	–	3.4	9.4				–
[7]	Same	–	–	11.10	–	0.110	–	–
[10]	Same	0.7–1.0	3.4	9.4	0.107	0.100	0.026	0.028
Calculation								
Our data, problem (15)	Circular	0.95	3.36	9.1	0.112	0.093	0.026	0.033
Our data, problem (18)	Same	0.85	3.40	9.36	0.110	0.100	0.027	0.036
[21]	Same	var	3.05	6.8	0.139	0.146	0.030	0.049

Calculation Results and Their Discussion. The above relations (13), (20), and (21) in combination with the analytical expressions for the functions f', h and their derivatives allow determination of all the main parameters of a vertical free-convective jet motion of a liquid over the basic (self-similar) section. But the domain of application of solutions (9), (12), (16), and (19) is limited by the corresponding values of σ_t, at which the equations allow integration by quadratures. In this connection, Martynenko and others [14, 16-19] have made an attempt to extend the domain of application of the analytical relations. However, the sought functions constructed within the framework of different approximate schemes and satisfying the general form of the analytical formulas were not sufficiently accurate, as revealed by analysis [20].

The four nonlinear two-point boundary-value problems were numerically solved by the Heming method by reducing (8), (11), (15), and (18) to the corresponding Cauchy problems. In calculations, the results of which are given in Tables 1 and 2, the turbulent Prandtl number ranged from 0.4 to 1.1, which is conditioned by the experimental data of [9, 10]. As follows from the tables, in the investigated σ_t range the characteristics of the jet flow undergo a monotonic change: $f'(0)$, $h(0)$, and $(uv)_m$ increase, while $\eta_{0.5u}$, $\eta_{0.5\Delta T}$, and $(vT)_m$ decrease. However, despite the general similarity, the dependences display different features: the use of formula (4) instead of (3) leads to a slower increase (decrease) in the main parameters. Noteworthy is the fact that σ_t has a pronounced effect on the thermal characteristics of the free-convective flow, while changes in the hydrodynamic quantities are manifested to a lesser degree. An important distinctive feature of problems (8) and (11) ((15) and (18)) is that the similarity of the profiles of the mean components of the vertical velocity and the excess temperature takes place at different numbers σ_t, which are 0.667 and 0.719 for a plane jet and 0.479 and 0.600 for an axisymmetric one. Hence, the use of more exact equations in the mathematical models to describe the sought characteristics makes the threshold number σ_t^* separating the conditions with $y_{0.5\Delta T} > y_{0.5u}$ from those with $y_{0.5\Delta T} < y_{0.5u}$, shift to higher Prandtl numbers.

We would also like to note the substantial difference of the semiempirical models for calculating free-convective flows from the similar models for forced flows: a characteristic feature of the results obtained within the framework of Eqs. (1) and (2) is the sensitivity to a change in the closing relations, especially for an axisymmetric jet. Next, the study of the adequacy and implementation of the models discussed has shown (Table 3) that theory adequately predicts special features and regularities of the free-convective liquid motion over the basic (self-similar) section with a slightly better quantitative correspondence for the axisymmetric jet. Disagreement with experiment for the peak values of $\langle u'v'\rangle/u_c^2$ and $\langle v'T'\rangle/u_c\Delta T_c$ has also been noted in calculations of vertical free-convective jets by more complicated models using the differential equations for Reynolds stresses and heat fluxes [21].

Thus, the results of a mathematical representation of free-convective jets in the context of models (1)-(6) rather satisfactorily agree with experiment [1-10] and are not worse than those obtained using substantially more complicated models [21].

NOTATION

u, v, vertical and horizontal components of the averaged velocity; T, averaged temperature; C_p, specific heat at constant pressure; ν_t, kinematic coefficient of turbulent viscosity; σ_t, turbulent Prandtl number; ρ, density; β, coefficient of volumetric expansion; g, free-fall acceleration; Q_0, flux of excess heat content; x, y, coordinates; u', v', components of the pulsation velocity; T', temperature pulsation; η, self-similar variable; $\Delta T = T - T_\infty$, excess temperature; $-\langle u'v'\rangle$, turbulent shear stress; $y_{0.5}$, jet halfwidth; b, width of the mixing layer; κ_0, experimental constant; F_0, Froude number. Subscripts: ∞, surrounding liquid; m, maximum value; c, axial line of the jet.

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